CENTRUL JUDEȚEAN de Excelență BISTRIȚA-NĂSĂUD

ADMITERE ÎN GRUPA DE EXCELENȚĂ
Disciplina: MATEMATICĂ
28 SEPTEMBRIE 2019
barem de corectare

XI

1). $\sqrt[n]{2ab-a}+\sqrt[n]{2bc-b}+\sqrt[n]{2ca-c}=\sqrt[n]{a(2b-1)}+\sqrt[n]{c(2c-1)}+\sqrt[n]{c(2a-1)}$

$\leq \sqrt[n]{ab^2}+\sqrt[n]{bc^2}+\sqrt[n]{ca^2}$……………………………………………………………………………………2p

$\sqrt[n]{ab^2}=\sqrt[n]{\underbrace{1\cdot 1\cdot \ldots \cdot 1}_{n-2}\cdot a\cdot b^2} \leq \frac{n-2+a+b^2}{n}$…………………………………………………………………2p

Deci $\sum \sqrt[n]{2ab-a} \leq \sum \sqrt[n]{2ab^2} \leq \frac{3(n-2)+(a+b+c)+(a^2+b^2+c^2)}{n} \leq$ ………………2p

$\frac{3n-6+\frac{a^2+b^2+c^2+3}{2}+a^2+b^2+c^2}{n}=\frac{3n-6+3+3}{n}$=3 …………………………………………………1p

2). a).$(\sqrt{2}-1)^x \overset{not}{=} t$

$t^2-2t-3=0;\ \Delta=16\quad t_1=1, t_2=3$…………………………………………………………2p

Revenim la notația inițială$(\sqrt{2}-1)^x=3$ ………………………………………………1p

b). Condiții:$\begin{cases}-x>0\\ -x\neq 1\end{cases} \Longrightarrow \begin{matrix}x<0\\ x\neq 1\end{matrix}$…………………………………………………………………………1p

Notăm $-x=t \Longrightarrow x=t^2$

$\log_3 t^2-2\log_t 9=2.$

$2\log_3 t-\frac{4}{\log_3 t}=2.\qquad \log_3 t \overset{not}{=} u$

u^2-u-
$2=0$……2p

u$\in\{-1;2\} \Longrightarrow t_1=\frac{1}{3}, t_2=9, x\in\left\{-9,-\frac{1}{3}\right\}$

$x=\frac{1}{\log_3(\sqrt{2}-1)}$……………………………………………………………………………………………1p

3). Fie a=z+$\frac{1}{z}$. Avem $a^3 = z^3 + \frac{1}{z^3} + 3\left(z + \frac{1}{z}\right)$..1p

$\Longrightarrow |a|^3 = \left|z^3 + \frac{1}{z^3} + 3\left(z + \frac{1}{z}\right)\right| \leq \left|z^3 + \frac{1}{z^3}\right| + 3\left|z + \frac{1}{z}\right| \leq 2 - 3|a|$..............................3p

$\Longrightarrow |a|^3 + 3|a| - 2 \leq 0$..1p

$\Leftrightarrow (|a| + 1)^2(|a| - 2) \leq 0$..1p

$\Leftrightarrow |a| \leq 2$..1p